CHAPTER 9

COLUMNS AND PILASTERS

9-1. Introduction. This chapter covers the design of reinforced masonry columns and pilasters. These structural elements are defined by their sectional configurations and heights. A masonry column is a vertical compression member whose height exceeds three times its thickness and whose width is less than one and one-half times its thickness. Figure 9-1 shows an isolated CMU column supported by a spread footing. A masonry pilaster is a vertical member of uniform cross section built as an integral part of a wall which may serve as either a vertical beam or a column or both. A pilaster projects from one or both faces of an unreinforced wall and usually projects in a reinforced wall. Figure 9-2 shows details of a typical reinforced CMU pilaster. Pilasters are designed similar to columns except that pilasters are laterally supported in the direction of the wall, while columns are typically unsupported in both directions. General design criteria, section properties, and allowable stresses used but not contained herein are covered in chapter 5.

9-2. Minimum requirements.

a. Limiting dimensions. The least nominal dimension of a masonry column or pilaster will be 12 inches, except that 8 inches minimum may be used provided the axial stress is not more than one-half the allowable axial stress.

b. Vertical reinforcement. The vertical reinforcement will not be less than $0.005A_g$ nor more than $0.04A_g$, where A_g is the gross area of the column or pilaster in square inches. A minimum of four No. 4 bars will be used. Bar lap splice lengths will be sufficient to transfer the design loads in the reinforcement, but not less than 48 bar diameters.

c. Lateral ties. All longitudinal bars for columns or pilasters will be enclosed by lateral ties. The minimum lateral tie size will be #2 bars for #7 or smaller vertical reinforcement and #3 bars for larger vertical reinforcement. The ties will be spaced not more than 16 bar diameters, 48 tie diameters, or the least nominal

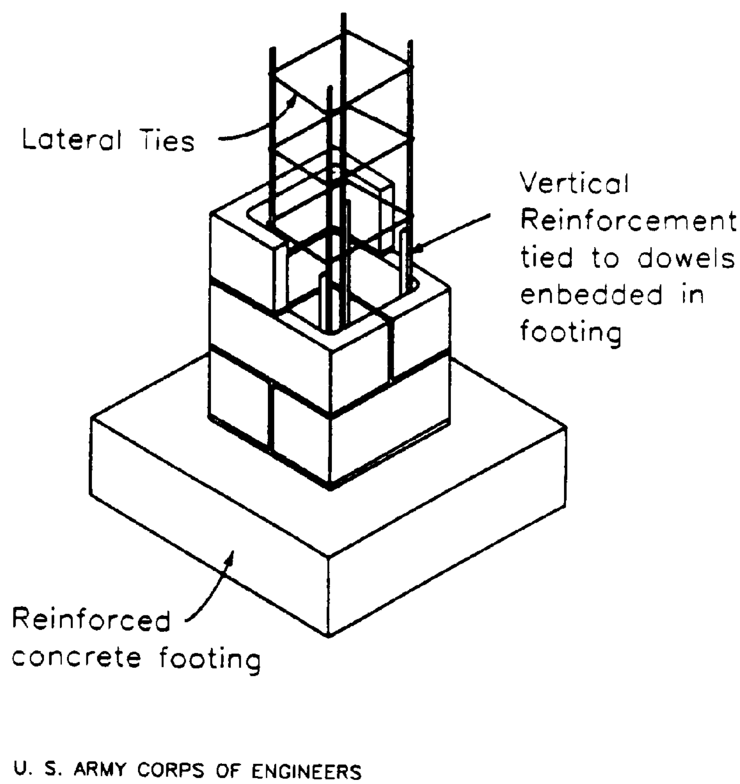

Figure 9–1. Isolated concrete masonry column.

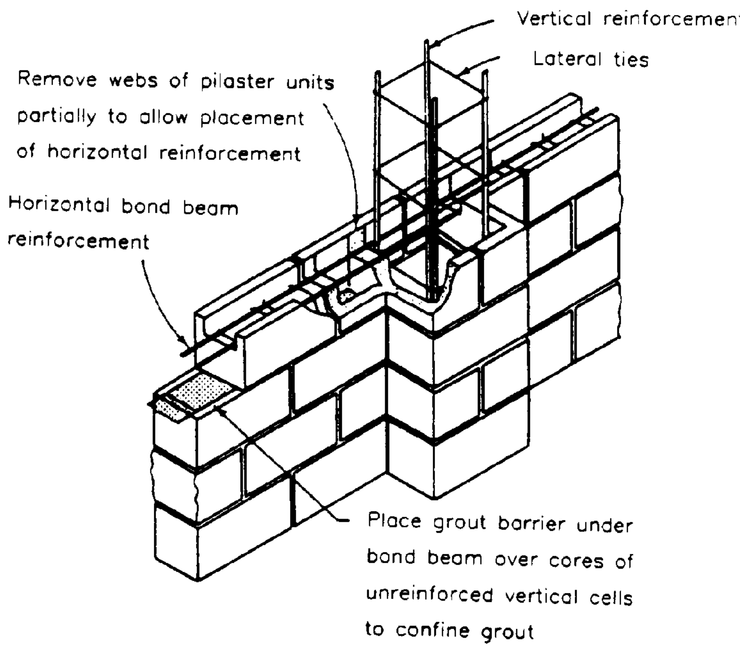

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Figure 9–2. Concrete masonry pilaster with continuous bond beam.

dimension of the column or pilaster. Lateral ties will be in contact with the vertical steel and not in the horizontal masonry bed joints. Ties will be arranged such that every corner and alternate longitudinal bar will be laterally supported by the corner of a lateral tie with an included angle of not more than 135 degrees or by a hook at the end of the tie. Bars which are unsupported by lateral ties will be spaced no further than 6 inches from a laterally supported bar. Hooks at the end of ties will turn a minimum of 135 degrees plus an extension of at least 6 longitudinal bar diameters, but not less than 4 inches at the free end of the tie. Lateral ties shall be placed not less than 1½ inches nor more than 3 inches from the top of the column. Additional ties of three #3 bars shall be placed within the top 5 inches of the column or pilaster.

9-3. Loadings. Columns and pilasters must withstand all applied vertical (axial) loads and in some instances, exterior pilasters (or columns when located between large doors, etc.) must also withstand lateral wind or seismic loads.

a. Axial loads. Vertical (axial) loads usually result from concentrated reactions imposed by beams, girders, or trusses which support dead and live loads from building floor or roof systems and rest on the column or pilaster. In addition, bending stresses in the column or pilaster will result when the axial loads are not applied at the centroid of the column or pilaster. The resulting bending moment will be the axial load, “P”, multiplied by the eccentrically of the load with respect to the centroid, “e”.

b. Lateral (wind) loads. Pilasters are in fact vertical wall stiffeners and will, due to this stiffness, attract lateral wind or seismic loads from the adjacent wall panels. When the adjacent wall is designed and detailed to span horizontally between pilasters, it must be assumed that the pilaster will carry the entire lateral load. However, when the wall panels contain a significant amount of vertical reinforcement as well as horizontal bond beams, lateral loads on the panels will be carried both vertically by the wall panel to the supporting roof or floor systems above and below and horizontally by the pilasters. The proportion of the lateral load transferred in each direction will depend upon the fixity or restraint at the panel edges, the horizontal to vertical span ratio, and the distribution of the applied loads. Curves are available in NCMA TEK No. 24 which provide coefficients that approximate the proportion of wind loads on wall panels that are transferred horizontally to the pilasters.

c. Combined axial and bending. Masonry columns and pilasters will be designed for the combined effects of axial compressive (or tensile) stresses and flexural stresses. All appropriate load combinations will be investigated. When the entire column or pilaster cross sectional area is in compression; i.e., the axial compressive stress is greater than the bending tensile stress; the entire cross section remains in compression and the section properties will be based upon what is termed the "uncracked section". This condition occurs when the virtual eccentricity, e_v, is less than or equal to $^1/_6$ of the thickness, t, of the member. ev is defined as the ratio of the moment, M, to the axial load, P. Figure 9-3 shows the uncracked section with three loading conditions where ev is less than or equal to t/6. When the flexural tensile stresses exceed the axial compressive stresses (e_v exceeds t/6) and the edge of the compressive stress block is at or outside the location of the reinforcing steel, the stress distribution is as shown in figure 9-4a. This condition, where the section is cracked but the reinforcing steel is not in tension, is not a consideration when the unity equation (equation 9-8) is used for design of combined stresses. It is, however, a point used in the development of the interaction diagram for a masonry pilaster or column. When the flexural tensile stresses exceed the axial compressive stresses, a portion of the cross section is cracked and the design cross sectional properties are based upon a reinforced masonry "cracked section" as shown in figure 9-4(b). Since it is assumed that the masonry will not resist tension, the reinforcement must resist all tensile forces. The design will be governed by the compressive stresses (axial and flexural) developed within the masonry section or by the flexural tensile stresses developed in the reinforcement. The combined loading effects will be considered in the design by using the basic unity interaction equation given later in this chapter.

d. Reaction location. Special consideration will be given to the effects created by the type and connection conditions of the members (beams, girders, trusses, etc.) supported by the masonry column or pilaster. If these members are not restrained against rotation, the resulting reaction will tend to move toward the edge of the support, increasing the eccentricity of the reaction. When a beam supported on a bearing plate is subject to rotation under loading, the vertical resultant reaction will be assumed at the third point of the bearing plate, as shown in figure 9-5(a). When a supported member displays very little rotation, due to its stiffness or continuity with other supported members, the load will be more uniformly distributed over the length of the plate, and the resulting reaction may be assumed to act at the center of the bearing plate, as shown in figure 9-5(b).

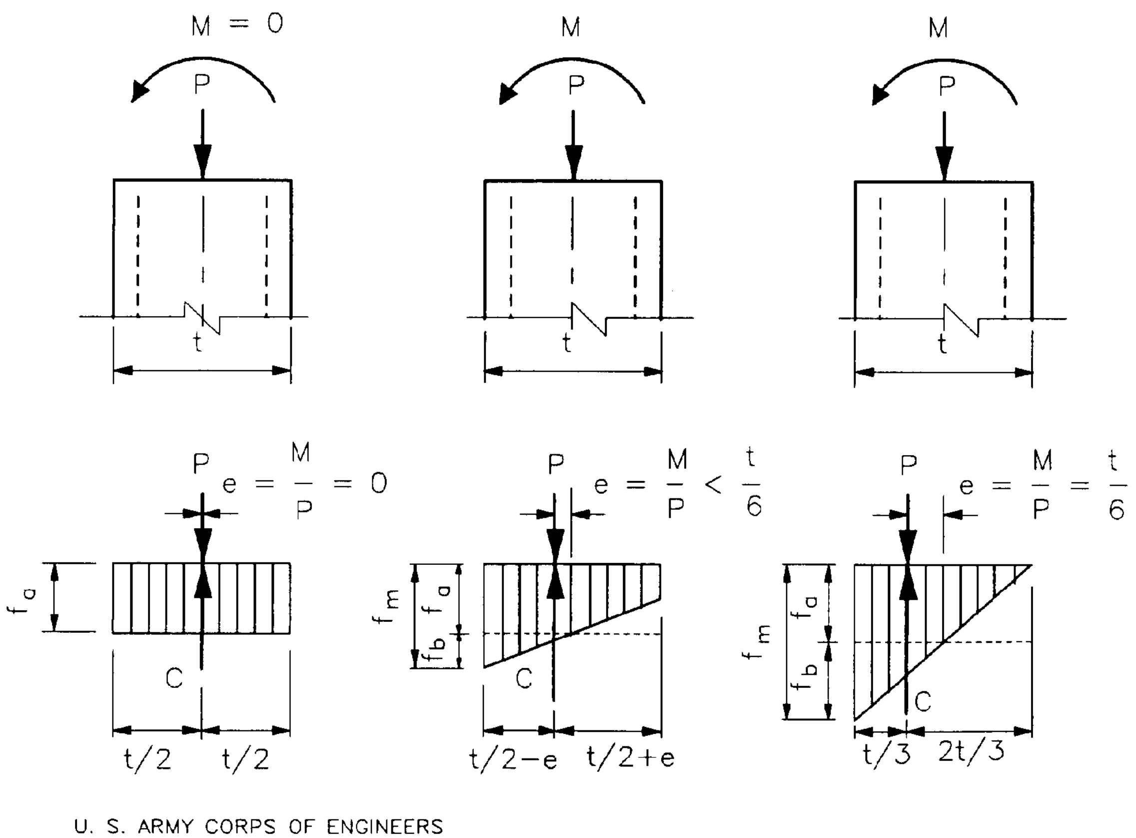

Figure 9–3. Uncracked section.

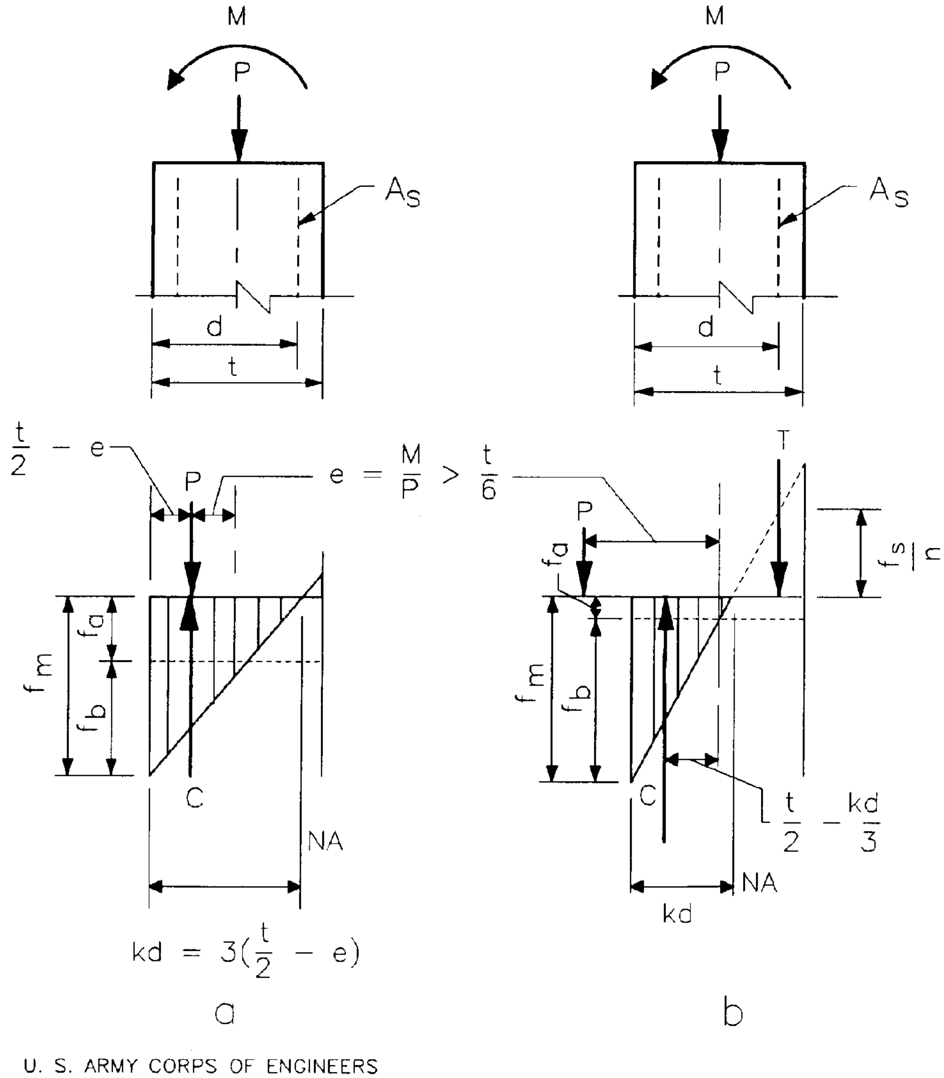

Figure 9–4. Cracked section.

9-4. General behavior. The behavior of columns and pilasters under axial loading is dependent upon. the cross sectional capacity of the column materials and the lateral stability of the column. Figure 9-6 illustrates this relationship between capacity and stability. In very short columns crushing failure occurs as the result of the load exceeding the ultimate material strength and stability does not become a design consideration. For most columns, inelastic deformation of the materials occurs on some portion of the column cross section before general column buckling occurs. Nonetheless, the allowable compressive stresses used during for design of the cross section are reduced to account for potential instability of the column. For long slender columns, elastic buckling failure will occur before any material reaches the yield state.

a. Effective height. The assumed behavior of columns and pilasters is a function of the slenderness of the member. The slenderness is expressed as the ratio of the effective height, h’, to the radius of gyration, r. h’ is the product of the clear height of the column, h, and the factor, K, which considers the effects of column end restraint and whether or not lateral deflection (sidesway) occurs at the top of the column. Values of K are provided in table 9-1. Since pilasters act as stiffening elements within a wall, they can be considered laterally supported in the direction parallel to the plane of the wall. However, slenderness effects must be considered in the direction perpendicular to the plane of the wall, and the design for that direction will be based on the effective wall height.

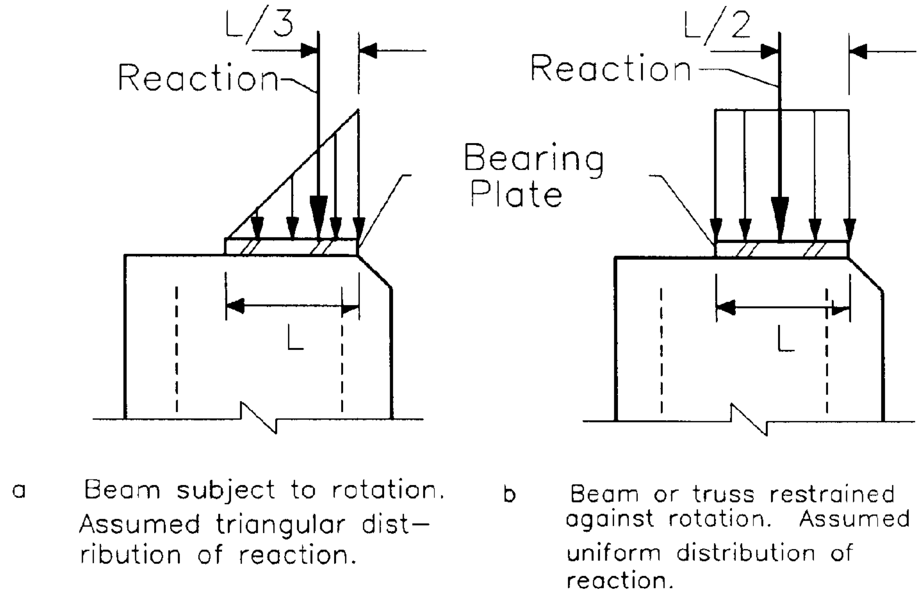

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Figure 9–5. Reaction locations of beams and trusses.

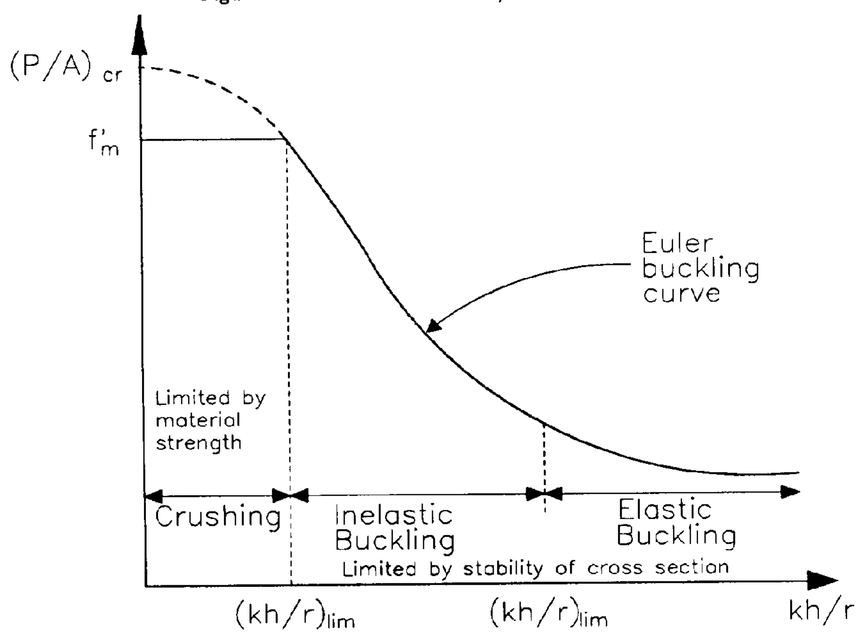

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Figure 9–6. Load carrying capacities of columns vs. Kh/r.

9-5. Design procedures.

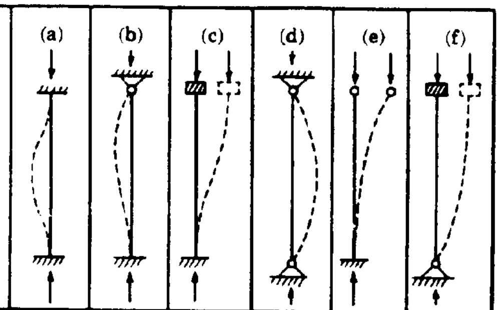
a. Axial compressive stress. In the design of masonry columns and pilasters, the compressive stress, f_a, is determined as follows:

$$f_a = \frac{P}{A_e} \text{ (psi)} \qquad \text{(eq 9–1)}$$

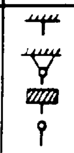
Where

P = The applied axial load, lbs.

A_e = The effective transformed area of the column (or pilaster) based on actual cross sectional dimensions, in^2.



	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0



End condition code	
	Rotation fixed and translation fixed
	Rotation free and translation fixed
	Rotation fixed and translation free
	Rotation free and translation free

Table 9–1. K factors for columns and pilasters.

$A_3 = [bt + (n - 1)A_s]$
b = The actual width of the column or pilaster, inches.
t = The least actual thickness of the column or pilaster, inches.
n = Modular ratio.
$= E_s/E_m$
E_s = The modulus of elasticity of the reinforcing steel, psi.
E_s = 29,000,000 psi.
E_m = The modulus of elasticity of the masonry, psi.
$E_m = 1000f'_m$ for CMU.
f'_m = The compressive strength of masonry, psi.
A_s = The cross-sectional area of reinforcing steel, in^2.

b. Allowable axial compressive stress. The allowable axial compressive stress for masonry columns and pilasters, F_a, is as follows:

$$F_a = [0.18f'_m + 0.65(p_g)(F_{sc})][R] \text{ (psi)} \qquad \text{(eq 9-2)}$$

Where:

p_g = The ratio of the cross-sectional area of the reinforcement to the gross area of the masonry section based on actual dimensions.
$p_g = A_s/A_g$
A_g = The gross area of masonry section based on actual dimensions, in^2.
F_{sc} = Allowable compressive stress in steel, psi.
$= 0.4f_y$
R = The stress reduction factor

$$R = 1 - \left[\frac{h'}{40t_n}\right]^3$$

t_n = Least nominal thickness of column or pilaster, inches.

Note. F_a may be increased by a factor of 1.33 when wind or seismic loads are considered.

In equation 9-2, the "$0.18f'_m$" is the allowable axial compressive stress provided by the masonry and the "$0.65p_gF_{sc}$" is the allowable compressive stress added to the section by the vertical reinforcement. The third term, R, reduced the stress to the point where buckling will not occur. R also accounts for the increase in moment due to lateral deflection or the P-Delta effect.

c. Flexural compressive stress. The computed flexural compressive stress, f_b, is computed as follows: For an uncracked section ($f_a \geq f_b$);

$$f_b = \frac{6M}{bt^2} \text{ (psi)} \qquad \text{(eq 9-3)}$$

Where:

M = The computed bending moment, inch-lbs.

For a cracked section ($F_a < f_b$);

$$f_b = \frac{2M}{bd^2jk} \text{ (psi)} \qquad \text{(eq 9-4)}$$

Where:

d = The effective depth of the flexural section measured from the extreme compression fiber to the centroid of the tension reinforcement, inches.

k = The ratio of the depth of the compressive stress to the depth, d.

j = The ratio of the distance between the centroid of the flexural compressive forces and the centroid of the tensile forces to the depth, d.

j = 1 - k/3.

d. Allowable flexural compressive stress. The allowable flexural compressive stress for masonry columns and pilasters, F_b, is determined as follows:

$$F_b = 0.33f'_m \text{ (psi)} \qquad \text{(eq 9-5)}$$

If f'_m = 1,350 psi; Then F_b = 0.33(1,350) = 450 psi

Note. F_b may be increased by a factor of 1.33 when wind or seismic loads are considered.

e. Flexural tensile stress. When tension reinforcing is required, the steel tensile stress, f_s, will be determined as follows:

$$f_s = \frac{M}{(A_{st})(j)(d)} \text{ (psi)} \qquad \text{(eq 9-6)}$$

Where:

A_{st} = The cross sectional area of the reinforcing steel that is considered in tension only, psi.

f. Allowable flexural tensile stress. The allowable tensile stress in reinforcing steel, F_s, when the yield strength of the reinforcement is equal to or greater than 60,000 psi is:

$$F_s = 24{,}0000 \text{(psi)} \qquad \text{(eq 9-7)}$$

Note. F_s may be increased by a factor of 1.33 when wind or seismic loads are considered.

g. Combined loading. Members subjected to combined axial and flexural stresses will be designed by the basic interaction equation as follows:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \qquad \text{(eq 9-8)}$$

Note. When F_a and F_b are increased by 1.33 for wind or seismic loads, the resulting design will be not less than the design determined using dead and live loads only. Load interaction methods based on accepted principles of mechanics may be used in lieu of equation 9-8.

h. Design considerations. The design of masonry columns or pilasters will consider the maximum loading conditions at the top of the member; at (or near) mid-height of the member, where the maximum bending will usually occur; and at the bottom of the member. Normally, masonry columns or pilasters will be given lateral support at the top by roof or floor system members and will be connected to the foundation below with reinforcing dowel bars. For this condition, the conservative assumption is made that the tops and bottoms of the members are simple supports. When other support or fixity conditions exist, calculations will be based on established principles of mechanics. At the top of a column or pilaster, a combination of the axial load, P, the eccentric moment, Pe, and any other loads and moments present will be considered. When determining F_a at the top of the member, the load reduction factor need not be considered. At (or near) the mid-height of a column or pilaster, a combination of axial load, P, the eccentric moment, approximately Pe/2, the lateral load moment, and any other loads and moments present will be considered. At mid-height, the P-Delta effect is at its maximum, thus F_a will be reduced by the load reduction factor, R. Since some value of lateral loads and lateral load moments will generally act in either direction on the member, the maximum combination of axial load, eccentric moment at the mid-height, moments created by lateral loads acting in either direction will be considered.

9-6. Design Example. The following design example illustrates a procedure for designing reinforced masonry pilasters. The design of reinforced masonry columns is very similar and will follow the same procedure except that stability in both directions must be considered in column design.

a. Given.

(1) Truss end reaction (P) = 40 kips
(2) Eccentricity (e) = 2 inches.
(3) Height of pilasters (h) = 16 feet
(4) Spacing of pilasters = 25 feet
(5) Lateral wind load on wall (w) = 20 psf
(6) The wind loading, w, acts both inward and outward.
(7) Masonry:
Type S mortar
$f'_m = 1350$ psi
$F_m = 0.33f'_m = 450$ psi
$E_m = 1000f'_m = 1{,}350{,}000$ psi
(8) Reinforcement:
$f_y = 60{,}000$ psi
$F_s = 24{,}000$ psi
$E_s = 29{,}000{,}000$ psi
(9) $n = \frac{E_s}{E_m} = \frac{29{,}000{,}000}{1{,}350{,}000} = 21.5$

b. Problem. Determine the pilaster size and vertical reinforcement.

c. Solution. The pilaster must be designed to resist the given eccentric axial load in combination with the lateral wind load. The design must be checked at the top and at the mid-height of the pilaster to determine the critical section. The design procedure is to select an economical pilaster cross section and check the selected section for the required loading conditions.

(1) Assumptions.

(a) The wall spans horizontally between pilasters and all lateral loading on the wall is transferred to the pilasters.

(b) The pilasters are pinned at top and bottom.

(c) The initial pilaster cross section will be 16 inches by 16 inches with 6-#9 vertical bars as shown in figure 9-7.

(2) Check the minimum and maximum reinforcement requirements.

A_s = The area of reinforcement, in^2
$= 6(1.00\ in^2) = 6.00\ in^2$
A_g = The gross area of the pilaster, in^2
$= bt$

Where:

b = The actual width of the pilaster = 15.62 in.
t = The actual thickness of the pilaster = 15.62 in.
$A_g = (15.62\ in)(15.62\ in) = 244\ in^2$

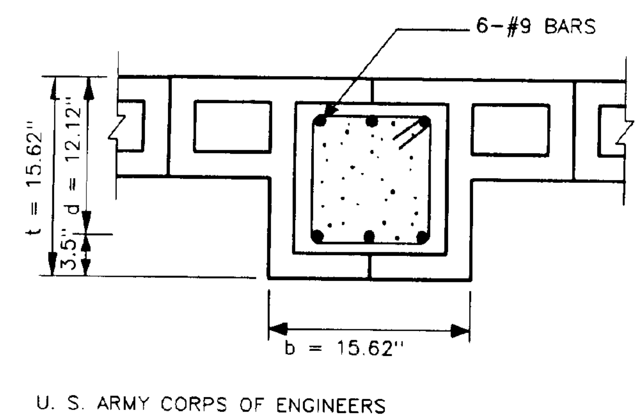


Figure 9-7. Trial section.

(a) The minimum reinforcement, A_{sMIN}, is determined as follows:

A_{sMIN} = 0. 005A_g = (0.005)(244) = 1.22 in^2

A_s = 6.00 in^2 > A_{sMIN}= 1.22 in^2

...O.K.

(b) The maximum reinforcement, A_{sMAX}, is determined as follows:

A_{sMAX} = 0. 04A_g = (0.04)(244) = 9.76 in^2

A_s = 6.00 in^2 < A_{sMAX} = 9.76 in^2

...O.K.

(3) Check the assumed pilaster for the given loadings at the top.

(a) The eccentric moment at the top, M_{eccT}, is determined as follows:

M_{eccT} = Pe = 40(2) 80 in-kips

(b) The axial compressive stress, f_a, is determined as follows:

$$f_a = \frac{P}{A_e} = \frac{P}{[bt + (n-1)A_s]} =$$

$$= \frac{40\text{ k }(1000\text{ lb/k})}{[244\text{ in}^2 + (21.5 - 1)6.00\text{ in}^2]} = 109\text{ psi}$$

(c) The allowable compressive stress, F_s, is determined as follows:

$F_a = [0.18f'_m + 0.65\, p_gF_{sc}]R$

Where:

$p_g = A_s/A_g$

= (6.00 in^2)/(244 in^2) = 0.0246

R = The stress reduction factor. *(Note.* R is one at top of pilaster since stability is not a consideration.)

F_a = [0.18(1350) + 0.65(0.0246)(24,000)]1.0 = 627 psi

f_a =109 psi < F_a = 627 psi

...O.K.

(d) The flexural compressive stress, f_b, is determined as follows (Note: Assume a cracked section):

$$f_b = \frac{2M_{eccT}}{bd^2jk}$$

Where:

d = 15.62 in - 3.5 in = 12.12 in; use d = 12 in.

A_{sT} = The area of the reinforcement that is in tension, which is 3-#9 bars.

A_{sT} = 3(1.00 in^2 = 3.00 in^2

p = A_{sT}/bd = 3.00 in^2/(15.62 in × 12 in) = 0.016

np = 21.5(0.016) = 0.344

$$k = \left[\,2np + (np)^2\,\right]^{1/2} - np$$

$$= \left[\,2(0.344) + (0.344)^2\,\right]^{1/2} - 0.344 = 0.554$$

$$j = 1 - k/3 = 1 - (0.554/3) = 0.815$$

$$f_b = \frac{2(80\text{ in-kips})(1000\text{ lb/kips})}{(15.62\text{ in})(12\text{ in})^2(0.815)(0.554)} = 158\text{ psi}$$

$$f_b = 158\text{ psi} < F_m = 450\text{ psi}$$

...O.K.

(e) Check combined axial and bending compressive stresses using the unity equation. When checking for flexural compression, the unity equation (equation 9-8) becomes:

$$\frac{f_a}{F_a} + \frac{f_b}{F_m} \le 1.0$$

$$\frac{109}{627} + \frac{158}{450} = 0.17 + 0.35 = 0.48 < 1.0$$

...O.K.

(4) Check the assumed pilaster for the given loadings at mid-height.

(a) The eccentric moment at mid-height, M_{eccM}, is determined as follows:

M_{eccM} = Pe/2 = 40(2)/2 = 40 in-kips

(b) The wind loading moment at mid-height, M_{wind}, is determined as follows:

$$M_{wind} = \frac{(w)(h^2)}{8}$$

$$= \frac{(20\ psf)(25\ ft)\ 16\ ft)^2\ (12\ in/ft)}{8\ (1000\ lbs/kip)} = 192\ in\text{-}kips$$

(c) The total moment at mid-height, M_{tot}, is determined as follows:
$M_{tot} = 40 + 192 = 232$ in-kips

(d) The axial compressive stress, f_a, is determined as follows (Note: The weight of the top half of the pilaster is added to P):

$$f_a = \frac{P}{A_e} = \frac{P}{[bt + (n-1)A_s]}$$

Where:

$$P = 40 + \frac{(244\ in^2)(16\ ft)(0.150\ k/ft^3)}{(2)144\ in^2/ft^2} = 42\ kips$$

$$f_a = \frac{42\ k\ (1000\ lb/k)}{[244\ in^2 + (21.5\ -\ 1)6.00\ in^2]} = 114\ psi$$

(e) The allowable compressive stress, F_a, is determined as follows:
$F_a = [0.18f'_m + 0.65\ p_gF_{SC}]R$
$=[0.18(1350) + 0.65(0.0246)(24{,}000)]R = 627(R)$ psi

Where:

$$R = \left[1 - \left[\frac{h'}{40t_n}\right]^3\right]$$

$$= \left[1 - \left[\frac{(16\ ft)(12\ in/ft)}{40(16\ in)}\right]^3\right] = 0.973$$

So;

$F_a = 627$ psi $(0.973) = 610$ psi
$f_a = 114$ psi $< F_a = 610$ psi

...O.K.

(f) The flexural compressive stress, f_b, is determined as follows (Note: Assume a cracked section):

$$f_b = \frac{2M_{tot}}{bd^2jk}$$

Where:

d = 15.62 in - 3.5 in = 12.12 in; use d = 12 in.
A^{sT} = The area of the reinforcement that is in tension, which is 3-#9 bars.
$A^{sT} = 3(1.00\ in^2) = 3.00\ in^2$
$p = A_{sT}/bd = 3.00\ in^2/(15.62\ in \times 12\ in) = 0.016$
$np = 21.5(0.016) = 0.344$

$$k = \left[2np + (np)^2\right]^{1/2} - np$$

$$= \left[2(0.344) + (0.344)^2\right]^{1/2} - 0.344 = 0.554$$

$$j = 1 - k/3 = 1 - (0.554/3) = 0.815$$

$$f_b = \frac{2(232\ in\text{-}kips)(1000\ lb/kips)}{(15.62\ in)(12\ in)^2(0.815)(0.554)} = 457\ psi$$

$$f_b = 457\ psi < F_m = (450\ psi)(1.33) = 600\ psi$$

...O.K.

(g) Check the adequacy of section using only the axial load and the moment created by its eccentricity (without the 1/3 increase in allowable stresses for wind loading). The flexural compressive stress, f_b, is determined as follows (Note. Assume a cracked section):

$$f_b = \frac{2M_{eccT}}{bd^2jk}$$

Where:

$$M_{eccT} = Pe/2 = 40(2)/2 = 40 \text{ in-kips}$$

$$f_b = \frac{2(40 \text{ in-kips})(1000 \text{ lb/kips})}{(15.62 \text{ in})(12 \text{ in})^2(0.815)(0.554)} = 79 \text{ psi}$$

$$f_b = 79 \text{ psi} < F_m = 450 \text{ psi}$$

...O.K.

(h) Check combined axial and bending compressive stresses using the unity equation. When checking for flexural compression, the unity equation (equation 9-8) becomes:

$$\frac{f_a}{F_a} + \frac{f_b}{F_m} \leq 1.0$$

$$\frac{114}{610} + \frac{79}{450} = 0.19 + 0.18 = 0.37 < 1.0$$

...O.K.

(i) Check combined axial and bending compressive stresses (including wind loading stresses) using the unity equation. When checking for flexural compression, the unity equation (equation 9-8) becomes:

$$\frac{f_a}{F_a} + \frac{f_b}{F_m} \leq 1.0$$

Where:

$$F_a = 610 \text{ psi } (1.33) = 811 \text{ psi}$$

$$F_m - 450 \text{ psi } (1.33) = 600 \text{ psi}$$

$$\frac{114}{811} + \frac{457}{600} = 0.14 + 0.76 = 0.90 < 1.0$$

...O.K.

(j) Check the tensile stress in the reinforcement (including wind loading stresses) using the unity equation.

The tensile stress in the reinforcement, f_s, is determined as follows:

$$f_s = \frac{M_{tot}}{(A_{sT})(j)(d)}$$

$$= \frac{(232 \text{ in-k})(1000 \text{ lb/k})}{(3.00 \text{ in}^2)(0.815)(12 \text{ in})} = 7907 \text{ psi}$$

The allowable tensile stress in the reinforcement, F_s, is:

F_s= 24,000 psi (1.33) = 32,000 psi

f_s = 7907 psi < F_s = 32,000 psi

...O.K.

d. *Summary*. The nominal 16-inch by 16-inch pilaster with 6-#9 reinforcing bars is adequate.